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Non-universal dynamics of staggered non-equilibrium particle systems and Ising chains

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Abstract. Non-universal dynamics is shown to occur in a one-dimensional non-equilibrium system of hard-core particles. The stochastic processes included are pair creation and annihilation (with rates ϵ and ϵ') and symmetric hopping rates which alternate from one bond to the next (p_A, p_B). A dynamical scaling relation between the relaxation time and the correlation length in the steady state is derived in a simple way for the case $\epsilon' > p_A \gg p_B \gg \epsilon$. We find that the dynamical exponent takes the non-universal value $z = 2\ln(\epsilon'/\epsilon)/\ln(p_B\epsilon'/p_A\epsilon)$.

For the special condition $\epsilon + \epsilon' = p_A + p_B$, where the stochastic system is in principle soluble by reduction to a free fermion system, the model is mapped to the Glauber dynamics of an Ising chain with alternating ferromagnetic bonds of values J_1 and J_2, in contact with a quantum thermal bath. The full time dependence of the space-dependent magnetization and of the equal time spin–spin correlation function are studied by writing the master equation for this system in the quantum Hamiltonian formalism. In particular, we obtain the dispersion relations and rigorously confirm the results obtained for the correlation length and for the dynamical exponent.

1. Introduction

This paper discusses non-universal critical dynamics of non-equilibrium particle systems, in which dynamic exponents depend on microscopic parameters (ratios of transition rates).

Non-equilibrium particle systems with stochastic dynamics show properties ranging from steady-state phase transitions to turbulence, shocks, and non-equilibrium analogues of strong fluctuations and critical behaviour [1, 2]. As with equilibrium critical phenomena, associated critical exponents are typically dependent on symmetries and dimensionalities and (for dynamics) conservation laws, but not on microscopic details. In particular, the dynamic exponent z in many simple systems describes diffusive relaxation towards equilibrium ($z = 2$), while in others it takes other parameter independent values, for example $z = 3/2$ in the one-dimensional noisy KPZ or Burgers equations [3–6].

Simple hard-core particle models of non-equilibrium stochastic dynamics can be mapped to quantum systems and, in particular, to spin models. This latter mapping is achieved either by a pseudo-spin representation of the particles or by associating particles with domain walls [7–11].

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However, it is well known that non-universal behaviour can exist in certain non-uniform spin systems with Glauber dynamics [12], such as that providing a simple model for freezing into non-equilibrium states and glassy dynamics [13–16]. Here the relationship $\tau \sim \xi^z$ between the relaxation time τ and the equilibrium correlation length involves a parameter-dependent dynamic exponent z [17–19]. This suggests that analogous non-universal behaviour can exist in non-uniform stochastic particle systems, and this is demonstrated here.

The model considered is defined on a one-dimensional lattice with L sites. We divide this into two sub-lattices, A and B, containing even and odd sites, respectively. Each site can be occupied by at most one particle. A pair of particles in adjacent sites can annihilate with rate ϵ'. A pair can be created on two empty adjacent sites with rate ϵ. A particle can hop to an empty nearest-neighbour site with rate p_A if it is in sublattice A and with rate p_B if it is in sublattice B.

The correlation lengths characterizing the steady-state particle separations on each sublattice can be obtained by dynamic balance conditions within a mean-field approximation. The characteristic time for diffusion across the smaller of the lengths, ξ, can then be estimated from the hopping process. This is particularly simple to do when, for example, the annihilation rate is much greater than the creation rate (when ξ is large) and one hopping rate is much greater than the other. This simple procedure already provides the non-universal behaviour of the dynamic exponent z.

For a special case where the rates satisfy one constraint, the quantum spin Hamiltonian representing the particle dynamics reduces to a free fermion form after a Jordan–Wigner transformation [20]. The alternative and simpler way used here of achieving an exact solution for this case is by mapping the stochastic particle system to the Glauber dynamics of an Ising model with alternating bonds J_1, J_2 [10, 21].

The correlation lengths and relaxation time readily provided by the simple argument in the particle picture thus translate, for the special rate relation, into results for the alternating bond Ising model, where they can be confirmed by exact calculation on the spin model.

The Glauber dynamics of this model is that resulting from coupling the static Ising system to a quantum thermal bath and considering the Van Hove limit of a weak interaction and very large times. By writing the master equation which describes the Glauber dynamics in a quantum Hamiltonian formalism, we can easily obtain the equation determining the time evolution of the equal-time spin–spin correlation function. From this quantity one can compute the equilibrium domain wall density in the Glauber problem which is equivalent to the steady-state density of particles in the stochastic particle dynamics [10, 21]. By taking the Fourier transform of this equation we obtain a system of four linear equations which determine the dependence of the eigenfrequencies with the wavevectors of the two spin excitations. The solution of the associated secular equation gives us the dispersion relation. The behaviour of the frequency at low wavevectors determines the critical exponent z. We fully confirm the results obtained previously.

The structure of this paper is as follows: in section 2, we define the dynamics of the particle system in terms of the constituent stochastic processes (diffusion, annihilation and creation). Using the simple mean-field and random-walk arguments referred to above, we obtain the relation between the relaxation time and the equilibrium correlation length and hence we determine the critical exponent z. In section 3 we present the Glauber dynamics of the alternating-bond Ising system, giving the rates of transition between the different configurations. We then go on to relate the spin-flip processes to domain wall hopping, creation and annihilation, which translate to the particle processes. In section 4, we write down the master equation in a quantum Hamiltonian formalism and derive the equation

giving the time evolution of the spin–spin correlation function. Hence, we obtain the dispersion relation and from it we can again compute z. Finally, in section 5 we present our conclusions.

2. The reaction–diffusion system and its non-universal dynamics

The hard-core particle dynamics is as specified in the introduction: a particle on the even sublattice may hop to its left nearest-neighbour site, provided it is empty, or to its right nearest neighbour (if empty) at rate p_A; the corresponding rate for hopping from the odd sublattice is p_B; in addition, pairs of particles can appear at (or annihilate from) adjacent empty (or full) sites at a rate ϵ (or ϵ'). These processes are depicted in table 1 (see later).

We now consider the description of the steady state. Here the particle densities ρ_A and ρ_B on the two sublattices will be uniform. Since the system is in equilibrium the processes of creation and annihilation have to balance each other. Therefore, one concludes that, provided the mean-field approximation applies, $\epsilon' \rho_A \rho_B = \epsilon\,(1-\rho_A)(1-\rho_B)$. The same must be true for the processes of diffusion. Hence $p_A\rho_A(1-\rho_B) = p_B(1-\rho_A)\rho_B$. One obtains from these relations $\rho_A \sim x_A/(1+x_A)$, where $x_A = (\epsilon p_B/\epsilon' p_A)^{1/2}$ and $\rho_B \sim x_B/(1+x_B)$, where $x_B = (\epsilon p_A/\epsilon' p_B)^{1/2}$. These densities allow us to extract two distinct separation lengths, the average distances ρ_α^{-1} between particles on each sublattice α.

We will be particularly interested in the critical situation where these two characteristic lengths are both large. That occurs when $\epsilon' \gg \epsilon$, giving large particle separations, or where $\epsilon \gg \epsilon'$ (large vacancies separations). For convenience we consider only the former case. It is also convenient to take $p_A \gg p_B$, to widely separate the two lengths. Both are still large provided $\epsilon'/\epsilon \gg p_A/p_B$. The shorter, controlling correlation length ξ is then

$$\xi \sim \rho_B^{-1} \sim x_B^{-1} = (\epsilon' p_B/\epsilon p_A)^{1/2}. \tag{1}$$

Now to determine the rate of approach to this steady state for $\epsilon' \gg \epsilon$ one uses a modification, appropriate to the particle dynamics, of an argument given by [22]. For large times the evolution of the system is determined by the limiting relaxational process, namely particles diffusing until they meet, when they annihilate. For situations close to the steady state, i.e. for long-time critical dynamics (starting from generic non-equilibrium initial states), the characteristic time involved is that for particles to diffuse across the shorter steady-state separation length, ξ, and to annihilate. Now to traverse two adjacent bonds both rates p_A and p_B enter and the effective diffusion rate is $4p_Ap_B/(p_A+p_B)$. Thus the characteristic time is $\tau \sim (\epsilon' p_B/\epsilon p_A)(p_A+p_B)/4p_Ap_B \propto \xi^z$ giving

$$z = 2\ln\left(\frac{\epsilon'}{\epsilon}\right) \Big/ \ln\left(\frac{\epsilon' p_B}{\epsilon p_A}\right) \tag{2}$$

in the regime of validity ($\epsilon'/\epsilon \gg p_A/p_B \gg 1$). This is a non-universal result, being dependent on ratios of rates. The above reasoning is based on the mean-field argument that we have used to determine ρ_A and ρ_B. Mean-field theory determines correctly the characteristics of the steady state, but it fails to predict the correct approach to the steady state [11]. This is due to the diffusion aspects of the problem, which are relevant in one dimension. The random-walk argument that we have given to determine τ incorporates these aspects. We will see in the next sections that the same results are obtained by an exact calculation based on an equivalent Glauber–Ising dynamics. That equivalence applies when the rates satisfy

$$\epsilon + \epsilon' = p_A + p_B. \tag{3}$$

This relation is also sufficient to reduce the quantum Hamiltonian representing the particle dynamics to a free fermion form, which gives another possible method for solving the particle dynamics exactly.

Finally, we note that the exponent z changes continuously with the value of the rates of the diffusion and reaction processes, as given by equation (2). A similar example of a critical exponent which changes continuously was found in the study of the dynamics of the q-state Potts model [23]. In this system, the fraction of spins which never flip up to time t, $r(q,t)$, decays like a power law $r(q,t) \sim t^{-\theta(q)}$ when the initial condition is random. The exponent $\theta(q)$ varies continuously with q.

3. An Ising system with generalized Glauber dynamics, and its relationship to reaction-diffusion processes

The Ising system considered is a spin-$\frac{1}{2}$ chain, where a spin on the even sublattice is coupled to its left nearest neighbour by a ferromagnetic Ising interaction of strength J_1 and coupled to its right nearest neighbour by an interaction of strength J_2 ($J_1 > J_2$) [17]. One can therefore write the Hamiltonian for this alternating-bond system in the form

$$H = -\sum_{l=1}^{L} J_l \sigma_l \sigma_{l+1} \tag{4}$$

where σ_l denotes the eigenvalues of $\hat{\sigma}_l^z$, $J_l = J_1$ for l odd and $J_l = J_2$ for l even. The transitions between different configurations of the system, i.e. between different sets of values of the spins, are of the Glauber type, i.e. the system evolves by single spin flips [12]. The probability per unit time that a spin will flip from its value to the opposite one is taken to be [17]

$$\omega(\sigma_l) = \tfrac{1}{2}\Gamma[1 - \tfrac{1}{2}\sigma_l(\gamma_l^{+}\sigma_{l-1} + \gamma_l^{-}\sigma_{l+1})] \tag{5}$$

with

$$\gamma_l^{\pm} = \tanh(K_{l-1} + K_l) \pm \tanh(K_{l-1} - K_l) \tag{6}$$

where $K_l = J_l/k_{\mathrm{B}}\mathrm{T}$. Notice that if we take $J_1 = J_2$ we will recover the usual Glauber rates. It can be easily verified that these rates satisfy detailed balance which is a sufficient condition for the steady-state distribution of the Ising system to be a Gibbs distribution. This dynamics can be derived [24] by coupling the Ising system, described by Hamiltonian (4) to an ensemble of free fermion baths with a grand canonical probability distribution, i.e. with a density matrix given (in the case of no interaction) by $\hat{\rho} = \exp[-\beta(\hat{H} - \mu\hat{N})]/Z$ where $\hat{H}$ and $\hat{N}$ are the Hamiltonian and the particle number operator of the free fermion bath, respectively, β is the inverse temperature and μ the chemical potential. The quantity Z is the grand partition function and is just a normalization factor. If the particle number of the fermion system is kept fixed, then the chemical potential is, for low enough temperatures, essentially equal to the Fermi energy of the system. The coupling is done via an interaction operator which couples the $\hat{\sigma}^x$ component of each spin to a thermal bath whose probability distribution is the one given above. The different copies of the thermal bath are totally uncorrelated. If one takes the joint limit of the interaction strength λ going to zero and $t \to \infty$ such that $t\lambda^2$ is a constant (limit of Van Hove) [25, 26] one obtains the transition rates given by (5), under the condition that the chemical potential (Fermi energy) of the fermion bath is much larger than the couplings J_1 and J_2. If that is not the case then the transition rates still have the form (5), but the parameter Γ is no longer a constant and

Table 1. Processes of transition for a spin in the even sublattice and the equivalent particle processes. The spin which is to flip is the central one. The coupling strength of the left bond is J_1, and of right bond J_2. An empty circle in the dual lattice is to be identified with a vacancy and a full circle with the presence of a particle. For a spin in the odd sublattice the rates of the last two processes should be interchanged.

Initial state	Final state	Transition rate
$\uparrow \circ \uparrow \circ \uparrow$	$\uparrow \bullet \downarrow \bullet \uparrow$	$\frac{1}{2}\Gamma[1-\tanh(K_1+K_2)]=\epsilon$
$\uparrow \bullet \downarrow \bullet \uparrow$	$\uparrow \circ \uparrow \circ \uparrow$	$\frac{1}{2}\Gamma[1+\tanh(K_1+K_2)]=\epsilon'$
$\uparrow \circ \uparrow \bullet \downarrow$	$\uparrow \bullet \downarrow \circ \downarrow$	$\frac{1}{2}\Gamma[1+\tanh(K_1-K_2)]=p_A$
$\uparrow \bullet \downarrow \circ \downarrow$	$\uparrow \circ \uparrow \bullet \downarrow$	$\frac{1}{2}\Gamma[1-\tanh(K_1-K_2)]=p_B$

depends on the value of the neighbouring spins. We will consider that we can take Γ as a constant.

It is well known that there is a duality relation between Glauber dynamics and the reaction-diffusion model of hard-core particles discussed above [10, 21]. To see this we take a given configuration of the Ising system and consider the lattice of sites located in the middle of the bonds between the Ising spins (dual lattice). If the neighbouring Ising spins have different signs then we place a particle at that site of the dual lattice. Otherwise we leave the site empty. In this way, we map domain walls in the Ising system to particles in the dual lattice. It can be shown that this mapping has a precise mathematical meaning [27]. The different possible processes of transition for a given spin, its translation in terms of particle processes, the rates associated and their relation are given in table 1. Notice that the sum of the rates of the first two processes is equal to the sum of the last two in agreement with (3). This shows that this system is in the class of systems that are integrable (the precise meaning of this word will be made clear later) through free fermions [7]. This allows the computation of correlation functions for a set of distributions of initial configurations [11, 27]. Here we will not pursue this point. If we now take the limit of low temperatures ($K_1 \gg K_2 \gg 1$) we see that the rates of pair annihilation (ϵ') and of diffusion from the strong bond to the weak bond p_A are equal to Γ. On the other hand, the rate of diffusion from a weak bond to a strong bond is $p_B \sim \Gamma \exp(-2(K_1 - K_2))$. Finally, the rate of pair creation is $\epsilon \sim \Gamma \exp(-2(K_1 + K_2))$. So we see that these two last processes are exponentially suppressed at low temperatures. Nevertheless, they have to be taken into account for a proper description of the steady state and diffusive dynamics, as we saw in section 2. We will see in the next section that the results given in section 2 may be obtained by an exact calculation.

4. The master equation in the quantum Hamiltonian formalism

In this section we will study the time evolution of the spin–spin correlation function. For that we need the full master equation for the evolution of the Ising system with rates given by (5). Since the master equation is a linear equation, a particularly convenient way to write it is to use an operator formalism which assigns to each configuration of Ising spins a vector $|\underline{n}\rangle$ in a Hilbert space. The probability distribution for the different configurations at a given time t can then be written as a state vector $|\Psi_t\rangle = \sum_{\underline{n}} P(\underline{n}, t)|\underline{n}\rangle$ [28], where $P(\underline{n}, t)$ is the probability of finding configuration $\underline{n}$ at time t and is a solution of the master equation. The set of different $|\underline{n}\rangle$ is supposed to be orthonormal and complete. One can then write the master equation in the compact form $\partial_t|\Psi_t\rangle = -\hat{T}|\Psi_t\rangle$, where $\hat{T}$ is a linear

and, in general, non-Hermitian operator that for two-state systems with local interactions can be written as a quantum spin Hamiltonian. The average values of quantities like the Ising spins can also be conveniently represented in this language as $\langle\sigma_j(t)\rangle = \langle s|\hat{\sigma}_j^z|\Psi_t\rangle$, where $\hat{\sigma}_j^z$ is the Pauli spin matrix at site j and $\langle s| = \sum_{\underline{n}}\langle\underline{n}|$. Substituting the state vectors and the operators by their definitions and using the orthonormality relations one sees that one obtains the usual definition of average over configurations. One also can, given the correspondence with the Schrödinger equation, define an Heisenberg representation of the operators by $\hat{A}(t) = \mathrm{e}^{\hat{T}t}\hat{A}\,\mathrm{e}^{-\hat{T}t}$, where $\hat{A}$ is a generic operator. These operators obey the equation of motion $\mathrm{d}\hat{A}/\mathrm{d}t = [\hat{T}, \hat{A}(t)]$. It should be stressed that this is only a convenient way to represent the master equation for this system and is not related to the intrinsic quantum dynamics of the Ising system which is given by the Hamiltonian (4). In fact, the dynamics is in this case generated by the interaction of the Ising spins with the thermal bath and cannot be deduced from the form of (4) alone. For the system that we have studied in the previous section the operator $\hat{T}$ has the form

$$\hat{T} = \frac{\Gamma}{2}\sum_{l=1}^{L}(1-\hat{\sigma}_l^x)\left[1-\frac{1}{2}\hat{\sigma}_l^z(\gamma_l^+\hat{\sigma}_{l-1}^z+\gamma_l^-\hat{\sigma}_{l+1}^z)\right] \tag{7}$$

as can be seen if we consider the matrix elements $\langle\underline{n}'|\hat{T}|\underline{n}\rangle$ and $\langle\underline{n}|\hat{T}|\underline{n}\rangle$, where $\underline{n}'$ is a configuration differing from $\underline{n}$ by the flip of a single spin σ_l, say. We obtain

$$\begin{aligned}\langle\underline{n}'|\hat{T}|\underline{n}\rangle &= -\frac{\Gamma}{2}\left[1-\frac{1}{2}\sigma_l(\gamma_{l-1}^+\sigma_{l-1}+\gamma_{l+1}^-\sigma_{l+1})\right]\\ \langle\underline{n}|\hat{T}|\underline{n}\rangle &= \frac{\Gamma}{2}\sum_{l=1}^{L}\left[1-\frac{1}{2}\sigma_l(\gamma_{l-1}^+\sigma_{l-1}+\gamma_{l+1}^-\sigma_{l+1})\right]\end{aligned} \tag{8}$$

where σ_l, etc, are the values of the Ising spins in configuration $\underline{n}$. The first relation gives (up to a minus sign) the transition rate between configurations $\underline{n}$ and $\underline{n}'$. The second relation gives the total rate of transition out of configuration $\underline{n}$. This agrees with the general definition of the $\hat{T}$ operator [29]. The form (7) and associated equations of motion are particularly convenient for they allow the calculation of multiple time spin–spin correlation functions. In the same way one can represent the master equation for the particle dynamics in terms of a $\hat{T}$ operator which is also a quantum spin Hamiltonian. The duality relation between these two models can be given a precise mathematical meaning by means of a similarity transformation between the operators of the two models [27]. In particular, the $\hat{T}$ operator of the generalized Glauber dynamics maps to the $\hat{T}$ operator of the particle system. This later operator can be expressed in terms of free fermions by a Jordan–Wigner transformation [20] and therefore the system is completely integrable. Many correlations functions relevant for the study of the particle dynamics and the associate Glauber problem can hence be obtained.

If one now considers the Heisenberg equation of motion for $\hat{\sigma}_j^z(t)$ and takes the average value on some arbitrary initial state $|\Psi\rangle$ in the way indicated above, one obtains the differential equation giving the time development of the average space-dependent magnetization [17]. This equation can be solved by defining the Fourier transforms m_k^+ and m_k^- of the magnetization at even and odd sites, respectively, with the wavevector k equal to $k = (2\pi/L)n$ with n an integer and such that k lies in the limits $-\frac{1}{2}\pi \leqslant k < \frac{1}{2}\pi$ (we have chosen the lattice spacing to be one). Notice that the Brillouin zone has been reduced to half of the size it would have if $J_1 = J_2$ since the lattice now has a periodicity of two. This procedure gives us a system of two equations coupling m_k^+ and m_k^- for every k.

The relaxational eigenvalues of this system are

$$\omega_k^{\pm} = 1 \pm [\cos^2 k \tanh^2(K_1 + K_2) + \sin^2 k \tanh^2(K_1 - K_2)]^{1/2} \tag{9}$$

where $\omega_k^{\pm}$ stands for the lower (acoustical) and upper (optical) branches of this dispersion relation. The late (critical) dynamics is determined by the low k modes of the acoustical branch ω_k^-. The characteristic length associated with the decay of the local magnetization in the steady state can be found from the value of $|k|$ $(=2\pi/\xi)$ making the right-hand side of (9) vanish. We, therefore, take the analytic continuation of (9) to the complex plane. We obtain the space decaying mode with infinite lifetime (which decays over a characteristic length ξ). For low temperatures, ξ is given by

$$\xi \sim \exp(2K_2). \tag{10}$$

Therefore, at low k and low temperatures one can write the dispersion relation as

$$\omega_k^- = 2\Gamma\xi^{-z}(1 + (k\xi)^2) \tag{11}$$

with ξ given by (10) and

$$z = 2 + \frac{J_1 - J_2}{J_2} \tag{12}$$

a result given in [17]. Now to determine the time evolution of the density of particles we need to compute the equivalent quantity in the Glauber language. This quantity is the domain wall local density [10] which is given by $\frac{1}{2}(1 - \langle\sigma_l(t)\sigma_{l+1}(t)\rangle)$. We, therefore, need the equation giving the time evolution of the spin–spin correlation function. Notice that these correlations do not involve only eigenmodes of (7) orthogonal to the slowly decaying ones, so their relaxation follows closely the slowest processes in the system. Using the transfer matrix formulation we can write

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle s|\hat{\sigma}_l^z(t)\hat{\sigma}_m^z(t)|\Psi\rangle = \langle s|[\hat{T}, \hat{\sigma}_l^z(t)\hat{\sigma}_m^z(t)]|\Psi\rangle$$

where $|\Psi\rangle$ is as above a general initial state. Substituting $\hat{T}$ by its expression given in (7) we obtain the equations

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\langle\sigma_l(t)\sigma_m(t)\rangle = {} & -2\Gamma\langle\sigma_l(t)\sigma_m(t)\rangle + \frac{\Gamma}{2}[\,\gamma_l^+\langle\sigma_{l-1}(t)\sigma_m(t)\rangle + \gamma_l^-\langle\sigma_{l+1}(t)\sigma_m(t)\rangle \\ & + \gamma_m^+\langle\sigma_l(t)\sigma_{m-1}(t)\rangle + \gamma_m^-\langle\sigma_l(t)\sigma_{m+1}(t)\rangle]\end{aligned} \tag{13}$$

if $l \neq m$ and the boundary condition $\langle\sigma_l(t)\sigma_l(t)\rangle = 1$. Since these equations are linear this boundary condition may be ignored. It can be later enforced by superposing different solutions [12]. To solve these equations we introduce Fourier transforms of the quantities $\langle\sigma_l(t)\sigma_m(t)\rangle$ when l and m are even, l is even and m is odd, etc. The two wavevectors k and k' which now appear are defined as above. We obtain a system of four linear equations. The associated secular equation has four solutions corresponding to having two excitations in the acoustic branch, one in the acoustic and one in the optical branch (two solutions) and two excitations in the optical branch. The frequency $\omega_{kk'}$ is given by

$$\omega_{kk'} = \omega_k + \omega_{k'} \tag{14}$$

where ω_k and $\omega_{k'}$ are given by one of the solutions in (9). If we now take the translational average of the correlation function $\langle\sigma_l(t)\sigma_{l+1}(t)\rangle$ we see that only the terms for which $k' = -k$ remain in the Fourier sum. Hence the decay of density of domain walls (density of particles) is determined by the acoustical branch of $\omega_{k,-k}$ for low k. At low temperatures we just obtain $\omega_{k,-k} = 2\omega_k^-$ with ω_k^- given by (11). The interparticle distance in the steady state is given by the value of k that makes $\omega_{k,-k}$ vanish. This coincides with (10). The relaxation

time τ is given by $\tau = \xi^z/4\Gamma$ ($\tau^{-1} = \omega_{0,0}$). This translates into (2) using the relationships between rates in the Ising and particle pictures (see table 1) in the low-temperature limit. So we recover the results obtained in section 2 in a rigorous way. It can also be seen from (14) that dynamical scaling still holds for this model. This is non-trivial, since as pointed out above there are two correlation lengths in the problem.

5. Conclusions

We have studied a generalized reaction-diffusion process, and the related model of generalized Glauber dynamics for a staggered Ising model. We obtained the following results.

(i) The dynamical exponent z relating the relaxation time to the equilibrium correlation length is non-universal, depending on the ratios of rates or the ratio between the couplings of the Ising model.

(ii) We found that dynamical scaling still holds, i.e. the dispersion relation still depends on a function of a single parameter $k\xi$ where k is the wavevector and ξ is the equilibrium correlation length.

It would be interesting to investigate the exact solution of this problem from the point of view of the particle dynamics, since one can diagonalize the dynamical operator $\hat{T}$ by using free fermions. Also it would be interesting to study if there is any change in the dynamical exponent z when one goes away from the free fermion condition. One would expect, based on the general argument given in section 2, that the relationship of z to the basic rates does not change provided that $\epsilon'/\epsilon \gg p_A/p_B \gg 1$. This means that in the long-time limit the system has a very low density of particles and therefore one should expect that the interactions are unimportant. Preliminary simulations seem to confirm this result [30].

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